Quaternion Based Robust Tracking Control of Kinematically Redundant Manipulators Subject to Multiple Self-Motion Criteria†

Hüsnü Türker Şahin, Ufuk Özbay and Erkan Zergeroğlu

Abstract— **In this study, a model based robust control scheme is developed for kinematically redundant robot manipulators that also enables the use of self motion of the manipulator to perform multiple sub-tasks in order to increase the manipulability and/or performance of the system. The proposed controller ensures uniformly ultimately bounded end-effector and sub-task tracking despite the parametric uncertainty associated with the dynamic model. A Lyapunov based approach has been utilized in the controller design and extension to a non minimum set of parameters for orientation representation has been presented to illustrate the flexibility of the approach. The capabilities and performance of the resulting controller is demonstrated by simulation results.**

Index Terms— **Robot control, kinematically redundant manipulator, self-motion control, robust/variable structure control.**

I. INTRODUCTION

Robotic manipulators are highly nonlinear multi-input multi-output systems subjected to uncertainties associated with their dynamics. Moreover, external disturbances are inevitable under practical operation conditions. For this reason an efficient tracking controller for robot manipulators should achieve sufficient robustness versus parametric uncertainties in the dynamics, as well as external additive disturbances. Kinematically redundant manipulators are complicated robotic systems with more degrees of freedom (DOF) than required to perform a task in the operation space. Owing to their extra DOF they can achieve much better performance in more dextrous operations, and/or have the increased flexibility for the execution of sophisticated tasks. Since the dimension (*i.e.*, n) of their link position variables is greater than the dimension (*i.e.*, m) of the operational space variables, the null space of their Jacobian matrix has a minimum dimension of $n - m$. Any link velocity in the null space of the manipulator Jacobian will not effect the operational space velocity and hence is referred to as *self-motion*. As stated in [1], [2], and [3], there are generally an infinite number of solutions for the inverse kinematics of a redundant manipulator. Thus, given a desired end effector trajectory, it is difficult to select a reasonable joint space trajectory that satisfies both control constraints (*i.e.*, stability and boundedness of all signals) and mechanical constraints (*i.e.*, singularities and joint limit avoidance). Therefore an efficient controller for kinematically redundant robot manipulators should be robust to parametric uncertainties and external disturbances, and achieve accurate end effector tracking, while reserving the self motion of the manipulator available for performance enhancement.

Many robotic manipulators are non-planar in geometry. For accurate operation in 3D task space, singularity free representation of the position and orientation (pose) information is also important. Frequently adapted orientation representation techniques such as Euler angles and Rodriguez parameters, have singularities for certain parts of operational space, which degrade controller operation. These singularities can be avoided by the utilization of 4 parameter based unit quaternion representations, however this slightly complicates the overall control design.

Due to the challenging nature of the fore-mentioned control design problem, many researches have proposed alternative solutions. To name a few; in [4], Khatib proposed a control scheme based on the dynamic model of a manipulator in Cartesian space and extended this result for redundant manipulators by using the pseudo-inverse of the Jacobian matrix. In [5], Seraji proposed the configuration control approach in which the end-effector motion in task-space is augmented by any $n - m$ dimensional additional tasks, such as optimization of kinematic and dynamic objectives or posture control. In [6], Hsu *et.al.* proposed a dynamic feedback linearizing control law that guarantees end-effector tracking and also provides control of redundant link velocities. In [7], Oh *et.al.* proposed a disturbance observer based robust controller that controls both the motion of the end effector and the null space motion of the redundant manipulator. In [8], Zergeroglu *et.al.* presented a model based controller that achieves exponential end-effector and sub-task tracking. The extensions for adaptive and model based output feedback type of controllers were also presented. However these controllers either required the exact knowledge of the dynamics, or assumed linearly parameterizable uncertain robot dynamics and the absence of external disturbances. From an examination of these previous research, we can group the control strategies for kinematically redundant manipulators in two categories. The first approach is based on extending the dimension of the operational space by incorporating additional constraints, so that the overall system becomes non-redundant. This approach usually introduces additional algorithmic singularities in the extended Jacobian matrix, and hence can cause the control input to become unbounded even away from manipulator singularities. The second approach is the generalized/pseudo-inverse based control formulations

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that utilize the pseudo-inverse of the manipulator Jacobian. This approach can not guarantee that the control remains bounded near the actual manipulator singularities, however it does not introduce further singularities. Considering more recent work, in [16], Braganza *et.al.* proposed an adaptive tracking controller for robot manipulators with both kinematic and dynamic uncertainties; and in [17], Tatlicioglu *et.al.* proposed an adaptive controller with some sub-task extensions, however most of these extensions required exact knowledge of the robot dynamics.

In this paper we have designed a robust controller for kinematically redundant robot manipulators based on generalized/pseudo-inverse formulation and the 4 parameter unit quaternion representation. Specifically we have developed a robust controller that achieves uniform ultimately bounded end-effector and sub-task tracking despite the parametric uncertainties associated with the dynamics and external additive disturbances. The proposed controller is also efficient for sub-task control. When compared with the previous controllers; with respect to [6] the controller proposed is robust to the parametric uncertainties in the robot dynamics. Compared to [7], the proposed controller does not introduce additional singularity issues due to the extended Jacobian formulation and with respect to [8] the proposed controller is capable of compensating a larger class of uncertainties. Also compared to [17] the proposed controller can compensate a larger set of uncertainties, includes a more realistic frictional term model with non-linearly parameterizable elements, and can perform multiple sub-task objectives without the need for system dynamical information.

The rest of the paper is organized as follows. Section II presents the kinematic and dynamic properties of redundant robot manipulators. Section III gives an introduction to quaternions. Section IV states the control objective and outlines the error system development. The controller design is presented in Section V. The use of multiple sub-task criteria for possible performance enhancement is presented in Section VI, followed by Simulation results in Section VII. Finally Section VIII contains the conclusions and future work.

II. ROBOT MODEL

A. Kinematic Model

The end-effector position and orientation in the operation space, denoted by $x(t) \in \mathbb{R}^m$, is defined as a function of joint position vector as [9]

$$x = f(q) = \begin{bmatrix} p(q) \\ \phi(q) \end{bmatrix} \tag{1}$$

where $f(q) \in \mathbb{R}^m$, $m \in \mathbb{Z}$ is the forward kinematic calculations, $q(t) \in \mathbb{R}^n$ denote the link position vector of an n-link manipulator $p(q) \in \mathbb{R}^l$ and $\phi(q) \in \mathbb{R}^{(m-l)}$ are the vectors representing the end-effector position, and orientation respectively and $l \in \mathbb{Z}$ is the size of the operational space.

From (1), the differential relationships between the end-effector and link position variables, is obtained as follows:

$$\begin{aligned} \dot{x} &= J(q)\,\dot{q} \\ \ddot{x} &= \dot{J}(q)\dot{q} + J(q)\ddot{q} \end{aligned} \tag{2}$$

where $\dot{q}(t)$, $\ddot{q}(t) \in \mathbb{R}^n$ denote the link velocity and acceleration vectors, respectively and $J(q) \triangleq \partial f(q)/\partial q, \in \mathbb{R}^{m\times n}$ is the Jacobian matrix of the manipulator. Note that for kinematically redundant manipulators the joint velocities may also be represented using (2) as follows

$$\begin{aligned} \dot{q} &= J^+\dot{x} + (I_n - J^+J)\,g \\ &= J^+\dot{x} + k\,(I_n - J^+J)\,[\nabla H(q)] \end{aligned} \tag{3}$$

where $J^+(q) \in \mathbb{R}^{n\times m}$ is the pseudo-inverse of the manipulator Jacobian and is defined in the following form [10]

$$J^+ = J^T\left(JJ^T\right)^{-1} \text{ such that } \quad JJ^+ = I_m. \tag{4}$$

In (3) $I_n \in \mathbb{R}^{n\times n}$ denotes the $n \times n$ identity matrix, $(I_n - J^+J)$ is the null space projection matrix. $J^+\dot{x}$ is the minimum norm joint velocity solution, $(I_n - J^+J)\,g$ is a homogenous solution of (3) in the null space of J orthogonal to $J^+\dot{x}$ and $g(t) \in \mathbb{R}^n$ is an auxiliary joint velocity vector which can be constructed to improve the performance of the manipulator according to the sub-task control objective (*e.g.*, mechanical limit avoidance, or obstacle avoidance). This possible performance enhancement is achieved by optimizing a proper performance criterion function, $H(q) \in \mathbb{R}$, where $\nabla H(q)$ is the gradient of $H(q)$ and k is a real valued scalar.

B. Dynamic Model

The dynamic model for an n-link, revolute, direct drive robot manipulator is assumed to be in the following form [9]

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) + \xi_d = \tau \tag{5}$$

where $M(q) \in \mathbb{R}^{n\times n}$ represents the inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ represents the centripetal-Coriolis matrix, $G(q) \in \mathbb{R}^n$ is the gravity vector, $F(\dot{q}) \in \mathbb{R}^n$ represents the friction effects, $\xi_d \in \mathbb{R}^n$ is a vector containing the unknown but bounded, additive disturbance effects and $\tau(t) \in \mathbb{R}^n$ is the torque input vector.

During the control development, we will make the commonly used assumption that the minimum singular value of the manipulator Jacobian, denoted by σ_m is greater than a known small positive constant $\delta > 0$, such that $\max\{\|J^+(q)\|\}$ is known a prior and all kinematic singularities are always avoided. We also note that since we are only concerned with revolute robot manipulators, we know that kinematic and dynamic terms denoted by $M(q)$, $C(q,\dot{q})$, $G(q)$, $J(q)$, and $J^+(q)$ are bounded for all possible $q(t)$.

III. UNIT QUATERNIONS FOR 3D ORIENTATION

In controller formulations for 3D task space operation, using a minimum set of 3-parameter representations, (ie. such as Euler angles) only forms a *local* parametrization of $SO(3)$ and exhibits singularities [13]. On the other hand quaternions provide a *global nonsingular* parametrization of $SO(3)$ at the cost of using 4-parameters for orientation. To utilize the advantages of nonsingular parametrization, we will follow a quaternion based approach.

Let the description of the manipulator's end-effector orientation with respect to its base frame is given by unit quaternion $\phi(t) = [\eta(t), \overrightarrow{\varepsilon}(t)]^T \in \mathbb{R} \times R^3$ with

$$\eta(t) \triangleq \cos\left(\frac{\varphi}{2}\right), \quad \overrightarrow{\varepsilon}(t) = [\varepsilon_1 \ \varepsilon_2 \ \varepsilon_3]^T \triangleq a \sin\left(\frac{\varphi}{2}\right) \quad (6)$$

where $\varphi(t) \in [0, 2\pi)$ and $a(t) \in \mathbb{R}^3$ are the Euler angle/axis parameters subject to the constraint $\overrightarrow{\varepsilon}^T\overrightarrow{\varepsilon} + \eta^2 = 1$. Note that given a unit quaternion representation, $\phi(t)$, the corresponding rotation matrix $R(q) \in \mathbb{R}^{3\times3}$ can be determined the formula [14]

$$R(q) = \left(\eta^2 - \overrightarrow{\varepsilon}^T\overrightarrow{\varepsilon}\right) I_3 + 2\overrightarrow{\varepsilon}\,\overrightarrow{\varepsilon}^T - 2\eta\overrightarrow{\varepsilon}^{\times}. \quad (7)$$

where the notation $\overrightarrow{a}^{\times}$, $\forall \overrightarrow{a} = [a_1 \ a_2 \ a_3]^T$, denotes the skew-symmetric matrix of the form

$$(\overrightarrow{a})^{\times} \triangleq \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}. \quad (8)$$

From quaternion algebra and (1), the differential relationship of (2) can be reformulated to have the form

$$\begin{bmatrix} \dot{p} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} J_p(q) \\ J_\phi(q) \end{bmatrix} \dot{q} \quad (9)$$

where $J_p(q) \in \mathbb{R}^{3\times n}$ and $J_\phi(q) \in \mathbb{R}^{4\times n}$ denote the position and orientation Jacobians respectively. The end effector angular velocity expressed in the manipulators base frame $\omega(t) \in \mathbb{R}^3$ is related to $\phi(t)$ via the following equation

$$\dot{\phi} = \frac{1}{2}B(\eta, \overrightarrow{\varepsilon})\omega \quad \text{with} \quad B(\eta, \overrightarrow{\varepsilon}) \triangleq \begin{bmatrix} -\overrightarrow{\varepsilon}^T \\ \eta I_3 - \overrightarrow{\varepsilon}^{\times} \end{bmatrix} \in \mathbb{R}^{4\times3}. \quad (10)$$

Applying $B^T B = I_3$ to (9) the following expression relating the end-effector velocity vector to the joint velocity vector can be obtained

$$\begin{bmatrix} \dot{p} \\ \omega \end{bmatrix} = J_q(q)\dot{q}, \quad (11)$$

where the end effector Jacobian matrix $J_q(q) \in \mathbb{R}^{6\times n}$ is:

$$J_q(q) \triangleq \begin{bmatrix} J_p(q) \\ 2B^T J_\phi(q) \end{bmatrix}. \quad (12)$$

IV. CONTROL OBJECTIVE

Our control objective is to design the control torque input signal $\tau(t)$ such that the robots end effector can follow a desired end effector pose signal as closely as possible. The designed control signal should also enable the redundancy of the manipulator to execute sub-tasks defined by at least one motion optimization measure (such as joint limit avoidance and/or obstacle avoidance). We will refer to the task space tracking as our "main" objective and enabling the use of manipulators redundancy as our secondary or "sub-task" objective.

A. Control Objective

To obtain a mathematical measure for our main control objective, we now define the desired end effector position and orientation and corresponding error signals. The end effector position tracking error is:

$$e_p = p_d - p \quad (13)$$

where $p_d(t) \in \mathbb{R}^3$ is the desired end effector position vector with the assumption that $p_d(t)$, $\dot{p}_d(t)$, and $\ddot{p}_d(t)$ are all bounded functions of time. To quantify the error between the actual and desired end effector orientations, we define the rotation error matrix $\tilde{R} \triangleq RR_d^T$ and the corresponding unit quaternion error representation is denoted by $e_\phi \triangleq [e_\eta, e_\varepsilon]^T$.

The angular velocity of the end effector frame with respect to the desired end effector frame $\tilde{\omega}(t)$ is defined as

$$\tilde{\omega}(t) = \omega - \tilde{R}\omega_d \quad (14)$$

where $\omega(t)$ was defined in (11) and $\omega_d(t)$ is the angular velocity of the desired end effector frame with respect to manipulator base frame. Similar to (10) the time derivative of $\phi_d(t)$ is related to $\omega_d(t)$ through

$$\dot{\phi}_d = \frac{1}{2}B(\eta_d, \overrightarrow{\varepsilon}_d)\omega_d \quad (15)$$

Based on the above definitions, our main control objective is to design the control torque input signal $\tau(t)$ to ensure that the end-effector position and orientation should track the desired end effector pose signals as close as possible. For our redundancy performance objective, we define the sub-task tracking error, $e_N(t) \in \mathbb{R}^n$ as follows [6]:

$$e_N = (I_n - J^+ J)(g - \dot{q}) \quad (16)$$

where $I_n \in \mathbb{R}^{n\times n}$ denotes the $n \times n$ identity matrix.

B. Error System Formulation

To obtain the end-effector position error formulation we take the time derivative of (13) to produce

$$\dot{e}_p = \dot{p}_d - \dot{p} \quad (17)$$

Similarly for the end-effector orientation error system formulation we utilized (10), (15), and (14), to obtain the time derivative of the unit quaternion $e_\phi(t)$ as follows

$$\dot{e}_\phi = \frac{1}{2}B(e_\eta, e_\varepsilon)\tilde{\omega} \quad (18)$$

where $B(\cdot) \in \mathbb{R}^{4\times3}$ was defined in (10). Using (10) and (15) the end effector orientation error can be formulated as

$$\dot{e}_\eta = -\tfrac{1}{2}e_\varepsilon^T\tilde{\omega}, \quad \dot{e}_\varepsilon = \tfrac{1}{2}\left(e_\eta I_3 - e_\varepsilon^{\times}\right)\tilde{\omega}. \quad (19)$$

At this point, following a similar approach given in [15], we define the following non negative scale function

$$V_o = \frac{1}{2}e_p^T e_p + (1 - e_\eta)^2 + e_\varepsilon^T e_\varepsilon. \quad (20)$$

Taking the time derivative of (20) along (17) and (19) then applying (14) we obtain

$$\dot{V}_o = e_p^T(\dot{p}_d - \dot{p}) + e_\varepsilon^T R_d^T(\omega - \omega_d) \quad (21)$$

After some mathematical manipulations (21) can be readjusted to the following form

$$\dot{V}_o = -e_p^T K_p e_p - e_\varepsilon^T K_o e_\varepsilon + \begin{bmatrix} e_p^T & e_\varepsilon^T \end{bmatrix} J_{ex} r_q \quad (22)$$

where the filtered tracking error like term r_q and the extended Jacobian matrix $J_{ex} \in \mathbb{R}^{6\times n}$ are defined as

$$r_q \triangleq (J_{ex})^{\#} \begin{bmatrix} \dot{p}_d + K_p e_p \\ -R_d^T\omega_d + K_o e_\varepsilon \end{bmatrix} + (I_n - J_q^+ J_q)g - \dot{q}$$
$$J_{ex} \triangleq \begin{bmatrix} I_3 & 0_{3\times3} \\ 0_{3\times3} & -R_d^T \end{bmatrix} J_q, \ (J_{ex})^{\#} \triangleq J_q^+ \begin{bmatrix} I_3 & 0_{3\times3} \\ 0_{3\times3} & -R_d \end{bmatrix} \quad (23)$$

with K_p, K_o of r_q being diagonal, positive definite gain matrices of proper dimensions, and $g \in \mathbb{R}^n$ is the component for sub-task objective. Pre-multiplying $r_q(t)$ defined in (23) with $(I_n - J_q^+ J_q)$, we obtain the following relationship

$$e_N = (I_n - J_q^+ J_q) r_q \quad (24)$$

which indicates when $r_q(t)$ is regulated the sub-task tracking error defined in (16) will also be regulated. Based on the structure of (20), (22) and the definition of $r_q(t)$ given in (23), the open loop dynamics of $r_q(t)$ can be obtained by pre-multiplying the time derivative of $r_q(t)$ by $M(q)$ and utilizing (5) to be in the following form:

$$M\dot{r}_q = \quad -Cr_q + \omega_{rq} - \tau \tag{25}$$

where the function $\omega_{rq}(t)$ is defined explicitly as follows

$$\begin{aligned}&\omega_{rq} \triangleq \\ &M(q)\frac{d}{dt}\left\{(J_{ex})^{\#}\begin{bmatrix} \dot{p}_d + K_p e_p \\ -R_d^T\omega_d + K_o e_\varepsilon \end{bmatrix} + (I_n - J_q^+ J_q)g\right\} \\ &+C(q,\dot{q})\left\{(J_{ex})^{\#}\begin{bmatrix} \dot{p}_d + K_p e_p \\ -R_d^T\omega_d + K_o e_\varepsilon \end{bmatrix} + (I_n - J_q^+ J_q)g\right\} \\ &+G(q) + F(\dot{q}) + \xi_d.\end{aligned} \tag{26}$$

Note that using the linearly parametrization property of the manipulator dynamics, ω_{rq}, can also be separated into LP and non-LP parts denoted by $W_q\theta$ and ξ_{rq} as follows:

$$\omega_{rq} = W_q\theta + \xi_{rq} \tag{27}$$

V. CONTROL DESIGN AND ANALYSIS

With the construction of the error dynamics as in (22) and (25), it is clear that a controller of the following from

$$\tau = W_q\hat{\theta} + K_q r_q + J_{ex}^T\begin{bmatrix} e_p \\ e_\varepsilon \end{bmatrix} + \frac{r_q\rho_q^2}{\|r_q\|\rho_q + \epsilon} \tag{28}$$

will ensure the globally uniformly ultimately boundedness of $e_p(t)$, $e_\varepsilon(t)$ and $r_q(t)$, where $\hat{\theta}$ is the vector of best guest estimates that were defined in (27), K_q is a diagonal positive definite gain matrix with proper size, ρ_q is the bounding function selected to satisfy

$$\rho_q \geq \left\|W_q\tilde{\theta}\right\| + \|\xi_{rq}\|. \tag{29}$$

Substituting (28) into (25) the closed loop error system for $r_q(t)$ can be obtained to be:

$$M\dot{r}_q = -Cr_q + W_q\tilde{\theta} + \xi_{rq} - K_q r_q - J_{ex}^T\begin{bmatrix} e_p \\ e_\varepsilon \end{bmatrix} - \frac{r_q\rho_q^2}{\|r_q\|\rho_q + \epsilon} \tag{30}$$

We are now ready to state the following theorem.

Theorem 1: The robust control law described by (28) ensures the globally uniform ultimately boundedness (UUB) of both the end effector position and orientation error signals in the sense that

$$\|e_p(t)\|, \|e_\varepsilon(t)\| \leq \|z_q(t)\| < \overline{d}, \quad t \geq 0 \tag{31}$$

where the composite state vector $z_q(t)$, is defined as

$$z_q(t) \triangleq \begin{bmatrix} r_q^T & e_p^T & e_\varepsilon^T \end{bmatrix}^T \tag{32}$$

In (31), $\overline{d} \in \mathbb{R}$ is a positive constant that defines the ultimate bound containing the end effector position and orientation error in the following form

$$\overline{d} = \sqrt{\frac{\left(\|z_q(0)\|^2 + \left(\frac{\epsilon}{\lambda_3} + \varphi\right)\right)}{\lambda_1}\exp\left(-\frac{\lambda_3}{\lambda_1}t\right) + \frac{\epsilon + \lambda_3\varphi}{\lambda_1\lambda_3}}, \tag{33}$$

where ϵ was introduced in (28), and the nonnegative functions $\varphi, \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}$ are defined as follows:

$$\begin{aligned}&\varphi = (1 - e_\eta)^2, \ \lambda_1 = \min\left\{\tfrac{1}{2}, \ \tfrac{m_1}{2}\right\}, \ \lambda_2 = \max\left\{1, \ \tfrac{m_2}{2}\right\}, \\ &\lambda_3 = \min\left\{\lambda_{\min}(K_q), \ \lambda_{\min}(K_p), \ \lambda_{\min}(K_o)\right\}.\end{aligned} \tag{34}$$

Proof: To prove *Theorem* 1, we start by defining the following non-negative scalar function

$$V = \frac{1}{2}r_q^T M r_q + V_o \tag{35}$$

where V_o was defined in (20). From direct application of properties of robot dynamics it is clear that (35) can be lower and upper bounded as follows

$$\lambda_1\|z_q\|^2 \leq V(z_q, t) \leq \lambda_2\|z_q\|^2 + \varphi \tag{36}$$

where $\lambda_1$and φ were defined in (34). Notice that the non negative function $\gamma_2(\|z_q\|) = \lambda_2\|z_q\|^2 + \varphi$ is radially unbounded, since $\gamma_2(0) = 0$ (As $z_q = 0$ implies $e_\eta = 1$) and $\lim_{z_p\to\infty}\gamma_2(\|z_q\|) = \infty$. After taking the time derivative of (35), substituting (22) and (30), cancelling common terms, and using the well-known skew symmetry property of inertia and coriolis matrices of robot dynamics, the following expression can be obtained

$$\begin{aligned}\dot{V} = \quad &-e_p^T K_p e_p - e_\varepsilon^T K_o e_\varepsilon \\ &+r_q^T\left(W_q\tilde{\theta} + \xi_{rq} - K_q r_q - \tfrac{r_q\rho_q^2}{\|r_q\|\rho_q + \epsilon}\right).\end{aligned} \tag{37}$$

Utilizing (29), (34) and the following result:

$$\rho_q\|r_q\| - \frac{\rho_q^2\|r_q\|^2}{\rho_q\|r_q\| + \varepsilon} = \rho_q\|r_q\|\left(1 - \frac{\rho_q\|r_q\|}{\rho_q\|r_q\| + \varepsilon}\right) \leq \varepsilon, \tag{38}$$

we can place the following upper bound on the right hand side of (37)

$$\dot{V} \leq -\lambda_3\|z_q\|^2 + \epsilon. \tag{39}$$

Utilizing left hand side of the inequality of (36)

$$\dot{V} \leq -\frac{\lambda_3}{\lambda_2}V + \frac{\epsilon + \lambda_3\varphi}{\lambda_2} \tag{40}$$

From (35) and (40), it is clear that the following condition holds

$$V(t) \leq V(0)\exp\left(-\tfrac{\lambda_3}{\lambda_2}t\right) + \tfrac{(\epsilon + \lambda_3\varphi)}{\lambda_3}\left(1 - \exp\left(-\tfrac{\lambda_3}{\lambda_2}t\right)\right) \tag{41}$$

Now we can utilize (34), (36) and (41) to obtain the result in *Theorem* 1. Hence when $r_q(t)$ is regulated inside an ultimate bound, from (24) the sub-task tracking signals will also approach to an ultimate bound in finite time. ⊡

Remark 1: Taking the limit of (34) as time approaches to infinity, we obtain

$$\lim_{t\to\infty}\overline{d} = \frac{\epsilon}{\lambda_3} + \frac{\varphi}{\lambda_1}, \tag{42}$$

which might mislead to a false conclusion, due to the term $\frac{\varphi}{\lambda_1}$*, that the size of the ultimate bound is not small enough for accurate tracking. However, we want to point out that as* $e_\varepsilon(t)$ *approaches inside the ultimate bound, the effect of this term on the ultimate bound also decreases since the value of* e_η *approaches to a value around 1.*

VI. MULTIPLE SELF MOTION PERFORMANCE CRITERIA OPTIMIZATION

The controller proposed Section V can achieve GUUB operational space tracking and still has the redundancy of the robot available to perform sub-tasks by utilization of the self motion of the manipulator. When the manipulator is subject to multiple performance criteria, the suitable selection and manipulation of the sub-tasks can be carried out according to the performance criterion function $H(q)$ defined in (3). An elegant way proposed in [18] suggests that, after the selection of proper performance criteria, their weighted sum can be formulated as an overall performance criterion as follows

$$H(q) = \sum_{i=1}^{s} \omega_i H_i(q) \tag{43}$$

where $H_i(q)$ is the scaler function expressing the ith desired performance criterion, ω_i's are positive valued, scaled functions representing the corresponding weights, and $s \in \mathbb{Z}_+$ is the maximum number of self motion (sub-task) criteria. Utilizing (43), the auxiliary joint velocity vector $g(t)$, defined in (3) can be expressed in the following form

$$g(t) \;=\; k\sum_{i=1}^{s} \omega_i \left[\nabla H_i\right] \;=\; k\left(\omega_1 g_1 + \omega_2 g_3 ... + \omega_s g_s\right) \tag{44}$$

where self motion control parameter $k \in \mathbb{R}$ was previously defined. Based on (44), multiple sub-task prioritization associated to the task can be achieved by adjusting the values of ω_i's that are subject to the following constraint

$$\sum_{i=1}^{s} \omega_i = P \tag{45}$$

where P is a real valued constant and is used in conjunction with the self motion control parameter k. Taken to the extreme, ω_i can be adjusted through out the execution of the task depending on the importance level of sub-task (e.g. giving more importance to joint limit avoidance then manipulability when the robot is very close to one of its joint limits).

A. Manipulability or Singularity Avoidance

By using the above principles, our first sub-task objective will be based on singularity avoidance for an nth DOF redundant manipulator, in addition to the main UUB tracking objective. Hence the performance criterion function will be selected according to the manipulability measure by [11]:

$$H(q) \;=\; \sqrt{det\left(JJ^T\right)} \tag{46}$$

where $J(q)$ is the manipulator Jacobian. This performance criterion function is based on purely robot kinematics. $H(q)$ declines to zero, when the manipulator approaches its singularities, thus it serves as a model on how far the manipulator is away from its singularities.

B. Manipulability and Joint Limit Avoidance

Nearly all robot manipulators are susceptible to joint limits as a result of their mechanical properties. That is the robot joint angles cannot be higher or lower respectively than a specified maximum angle $q_{i_{max}}$ and a minimum angle $q_{i_{min}}$. For this reason another important sub-task for redundant manipulators is joint limit avoidance. As a related measure, we have adopted the following function of joint variables $q_i(t)$ and limits from [20]:

$$H(q) \;=\; \prod_{i=1}^{n} 4\frac{(q_{i_{max}} - q_i)(q_i - q_{i_{min}})}{(q_{i_{max}} - q_{i_{min}})^2}, \tag{47}$$

where n is the number of robot joints. This function is the sum of contributions from all joint limits and automatically gives higher weight to the joints further away from their bounds. Accordingly, each term of the summation (47) takes the value 1 when the robot is at the furthest angle from the associated upper and lower joint limits, and declines to zero at the limits. Hence this function also offers a normalization on the variations of robot motion.

The advantage of employed multi-performance criteria method is that each sub-task objective can be combined in a weighted sum to from an overall objective. Accordingly the two sub-objectives of manipulability and joint limit avoidance can be combined via (44) to form the multi-subtask criteria. Thus if these are of equal importance in the robot operation range, their sums can be weighted equally with 0.5 value as follows:

$$\begin{aligned} H(q) \;=\;& 0.5\left(\sqrt{det\left(JJ^T\right)}\right) \\ &+ 0.5\left(\prod_{i=1}^{n} 4\frac{(q_{i_{max}}-q_i)(q_i-q_{i_{min}})}{(q_{i_{max}}-q_{i_{min}})^2}\right). \end{aligned} \tag{48}$$

If the operation of the robot is more under the influence of one singularity than the other these weight can be increased or decreased accordingly.

VII. SIMULATION RESULTS

To illustrate the performance of the proposed controller, an initial set of simulation results will be presented. In these simulations we have utilized the model of a Puma 560 type robot manipulator with the dynamical terms as in [19]. The desired task-space position trajectory, was selected to be:

$$\begin{bmatrix} x_d(t) \\ y_d(t) \\ z_d(t) \end{bmatrix} = \begin{bmatrix} 0.1\ \sin(0.5t)(1-e^{-0.1t^3}) + 0.17 \\ 0.1\ \sin(0.5t)(1-e^{-0.1t^3}) - 0.62 \\ 0.014 \end{bmatrix} [m] \tag{49}$$

and the orientation trajectory was generated by the following angular velocity components obtained from (15) as:

$$\begin{aligned} \omega_{d_{x,y,z}}(t) \;=\;& 0.02\cos(0.5t)(1-e^{-0.1t^3}) \\ &+ 0.01\sin(0.5t)(1-e^{-0.1t^3}) \end{aligned} \; [rad/s] \tag{50}$$

with the initial quaternion, $q_d(0) = [-0.5, 0.5, 0.5, 0.5]^T$. The controller parameters are tuned to the following values for satisfactory controller performance:

$$\begin{gathered} K_p = diag\left\{\ 18 \quad 18 \quad 20\ \right\},\ K_o = diag\left\{\ 15 \quad 17 \quad 17\ \right\} \\ K_q = diag\left\{\ 90 \quad 450 \quad 280 \quad 28 \quad 17 \quad 12\ \right\} \\ \rho = 8,\ \varepsilon = 0.05 \end{gathered} \tag{51}$$

Figure 1 depicts the position errors, where all terms quickly converge to zero after a short initial transient period. The orientation errors also show similar characteristic as in Figure 2 with the real part of error e_η tending to unity and the imaginary components of e_ε to zero in parallel with the stability properties of the designed controller. The torques

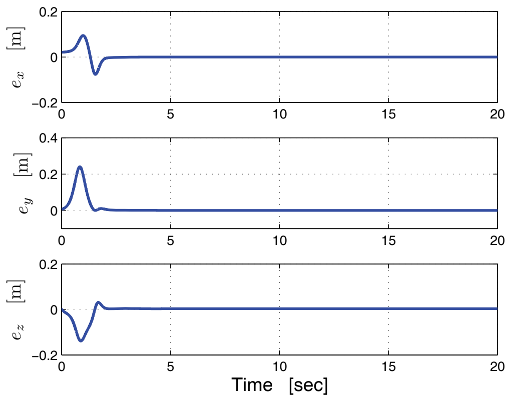

Fig. 1. Position errors for the designed controller

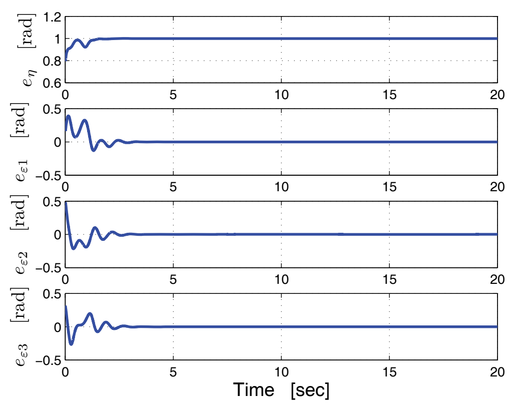

Fig. 2. Orientation errors for the designed controller

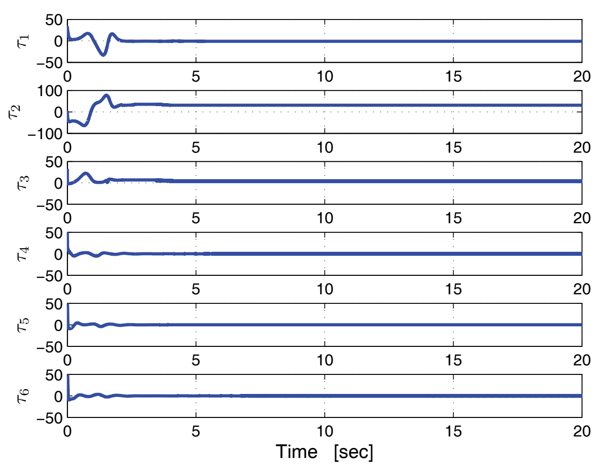

Fig. 3. Torques for the designed controller.

are in Figure 3, which are bounded functions of time. These results verify the stability and performance of the proposed controller.

VIII. Conclusion and Future Work

In this work, we designed a nonlinear robust controller that achieves globally uniformly ultimately bounded end-effector position and orientation and sub-task tracking, despite the presence of uncertainties in dynamic model and external disturbances. The control strategy is based on unit quaternion orientation representation, uses the pseudo-inverse of the manipulator Jacobian and does not require computation of the inverse kinematics. Thus it results in non-singular controller inputs, improving performance of the design. More importantly, the controller does not place any restriction on the self-motion of the manipulator, thus the extra degrees of freedom are available for sub-tasks like maintaining manipulability, avoidance of mechanical limits and obstacle avoidance, without the need for system dynamical parameters. The stability and effectiveness of the designed controller is verified by simulation results.

Future work on this project will be on application of this controller to a higher-DOF robot model for achieving multi-sub task performance criteria optimizations.

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