A Computationally Efficient Path Planner for a Collection of Wheeled Mobile Robots with Limited Sensing Zones

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Abstract— **Path planning for formations of wheeled mobile robots (WMRs) has been an active research topic of robotics in the recent years. The methods of route generation for WMRs usually utilize complex algorithms, which require global information of the workspace or mapping of the nearby environment. Hence they are not very efficient for formation based motion of large collections of small entry-level agents. In this work a path generation method is developed for synthesizing non-holonomic paths for small unicycle WMRs, and then is integrated into simple geometrical formations for extension to navigation of relatively large groups of small agents. Our technique is based on a computationally low-cost algorithm; and works effectively for transfer of many robots in obstacle cluttered environments. The efficiency of our technique is verified by simulation results.**

I. INTRODUCTION

Smooth path generation for nonholonomic single and multiple wheeled mobile robots (WMRs) is being researched significantly in the recent years. The nonholonomic constraints of wheeled systems impose difficulties for effective path planning and the presence of obstacles in practical environments adds to the complexity of the problem. Many alternative solutions have been proposed on route planning for single WMRs, ranging from curvature based shortest path graph search methods [1]–[4], to the application of potential function based methods [5]–[8]. Recent methods such as [10], [11] offer more efficient path planning in obstacle cluttered environments. However, the previous approaches in general use complex and computationally costly algorithms, some of which utilize expensive laser scanners for the low level path planning task. This inevitably limits the scarce robotic resources for more high level and useful tasks such as search, rescue or transportation tasks, and thus is a major drawback for wide scale application of swarms. The general strategy of swarm robotics is to keep the hardware and software based costs associated with the individual members to minimum, so that there is enough resources for planning and managing the formation framework for the aforementioned specialized tasks. Various different approaches have been proposed for the collective motion of robots [13]–[20]. However, most of these techniques are confined to a small number of robots as even tasks like obstacle avoidance and keeping correct margins between the agents require significant computational cost.

In this paper we first present an efficient approach for the WMR path planning problem in obstacle ridden environments. We then use this technique in conjunction with simple geometrical formations for the route generation of a relatively large collection of robotic agents. Our single robot planner is based on treating the WMRs as a two axle device, the back-wheel of which forms a readily nonholonomic reference trajectory for unicycle robots to follow as the front axle is steered in the direction of the WMR target. A simple and computationally low cost obstacle avoidance scheme is integrated with this direct steering mechanism to achieve trajectory generation under minimum sensing conditions, ie. in the presence of only a small number of ON-OFF sensors providing a very small sensing zone. The path planner enters obstacle avoidance mode only if a blocking body is sensed; and hence dynamically re-adjusts the drag force applied to the front axle to keep away from the obstacle. These comprise an effective dynamical path planner for single WMRs with limited sensing abilities in environments of mobile as well as fixed obstacles.

The utilized geometric formation is steered nonholonomically from its initial position to desired target in a similar way to the single WMR front steer mechanism. Each bicycle agent reference robot tracks its associated reference point on this formation, thereby synthesizing smooth trajectories for unicycle WMRs to follow. When obstacles are encountered, the robots deviate from these tracks according to the obstacle avoidance rules of the single WMR planner. After the effects of these obstacles are over, the obstructed agents readjust their direction towards the formation zone. The rendezvous with their locations on the geometrical frame is in a multi-phase period: In the first phase, the agent steer force is towards a virtual leader like general steer point at the front part of the formation. After a margin encirculating the collection zone is reached, this force gradually changes towards the actual reference point of the agent. The main advantage of this technique is it enables more accurate and less computationally expensive trajectory generation for relatively large agent collections in very obstacle dense environments. By this way real-time path generation has been achieved for groups in excess of 20 robots in obstacle dense environments such as complicated tunnels, using Matlab/Simulink platforms on a standard Windows XP based notebook.

The rest of the paper is organized in the following manner:

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In Section II an introduction to the kinematic WMR model is given, while Section III presents a brief statement of the overall problem. In Section IV the components of the overall path planning algorithm are discussed. Simulation results are given in Section V, and Concluding remarks are summarized in Section VI.

II. KINEMATIC MODEL

The path planner for single WMRs utilizes the following kinematic model of a unicycle WMR, also referred to as the kinematic wheel, [12]:

$$\dot{q} = S(q)v \tag{1}$$

where $q(t) \in \mathbb{R}^3$ denote the system pose vector such that:

$$q = \left[\begin{array}{ccc} x_r & y_r & \theta_r \end{array}\right]^T . \tag{2}$$

Here $x_r(t), y_r(t)$ represent the position of the WMR center of mass (COM) and $\theta_r \in \mathbb{R}^1$ is the robot orientation angle in the 2-D configuration space. The matrix $S(q) \in \mathbb{R}^{3\times 2}$ in (1) is defined by

$$S(q) = \left[\begin{array}{cc} \cos\theta_r & 0 \\ \sin\theta_r & 0 \\ 0 & 1 \end{array}\right], \tag{3}$$

and the velocity vector of the robot $v(t) \in \mathbb{R}^2$ is given by

$$v(t) = \left[\begin{array}{cc} v_l & \dot{\theta}_r \end{array}\right]^T, \tag{4}$$

with v_l the linear and $\dot{\theta}_r$ the angular velocity components of the robot, related to cartesian velocities $\dot{x}_r$ and $\dot{y}_r$ by:

$$\dot{x}_r = v_l \cos\theta_r, \quad \dot{y}_r = v_l \sin\theta_r . \tag{5}$$

For accurate motion of WMRs, the nonholonomic motion constraint of pure rolling and no slipping must be satisfied:

$$\dot{x}_r \sin\theta_r - \dot{y}_r \cos\theta_r = 0. \tag{6}$$

III. THE PROBLEM FORMULATION

The main objective of this work is to generate nonholonomic and non-colliding paths for the members of a large group of small unicycle WMRs moving in a formation governed by simple geometrical techniques. These tasks should be achieved in unstructured environments cluttered with obstacles with no access to global information, and real time operation is necessary for the implementation of the technique. Therefore this multiple-robot technique should be based on a computationally efficient and accurate single robot path planner. This low level planner is realized via the 2-axle reference robot with the front axle COM position having the values $P = [\,x_e,\ y_e\,]^T \in \mathbb{R}^2$ as in Figure 1. If this robot is steered from its front axle position P to a desired front end location $P_d = [\,x_e,\ y_e\,]^T$, its back wheel follows tracing a readily nonholonomic trajectory form its current position C_r to the desired target C_d. Any obstacles that may be encountered can be avoided via suitable orientation changes by altering the applied front steer force to keep the robot direction away from the blocks. Geometrical formation based approaches can be integrated with this simple planner to comprise a computationally efficient and effective flock planner in obstacle ridden environments.

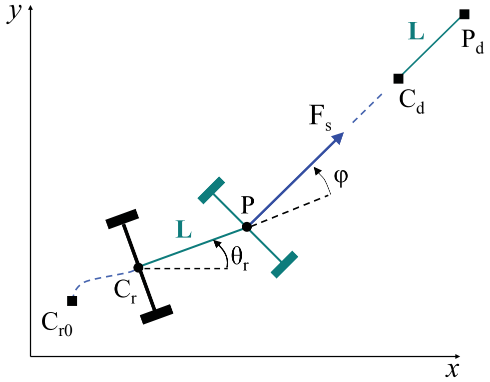


Fig. 1. The bicycle reference robot employed for steering WMRs.

IV. THE PROPOSED PATH PLANNING ALGORITHM

We can consider our path planner in to two main parts: the first is a single WMR planner for collision free trajectories in environments with fixed and possibly mobile obstacles; and the second is a simple geometrical structure for extension to multi-robot transportation. These techniques are summarized in the next two sub-sections.

A. The Single Robot Path Planner

1) Nonholonomic Steering Towards a Desired Target Location: The general kinematics modelling the relation between the front axle steer force F_s and nonholonomic trajectory generation via the back axle depicted in Figure 1, are extended from (1) and (5) as follows:

$$\begin{array}{ll} \dot{x}_r = v_l \cos\theta_r, & \dot{y}_r = v_l \sin\theta_r, \\ \dot{\theta}_r = \frac{v_l}{L}\sin\varphi, & v_l = F_s \cos\varphi \end{array} \tag{7}$$

Here the new terms are the linear velocity v_l and the angular velocity $\dot{\theta}_r$, as a function of the steering force F_s; the steering angle φ; and L (which is the distance between the point P on the synthetic front axle and the back axle COM C_r). Accordingly if a force F_s is applied to center point of front axle P in a direct linear path to a final end point P_d, this path passes through the specified desired position of the reference wheel, C_d. Thus as the front axle reaches P_d, the back wheel center should terminate about C_d, thereby generating a smooth reference trajectory from the initial position to the desired target.

The forcing function applied to the front axle center, P, is constructed to have the following form:

$$F = K\frac{e}{\sqrt{\|e\|^2 + \varepsilon}} + B\dot{e}. \tag{8}$$

where $F = [\,F_x\,,\ F_y\,]^T$ represent the forces in the x and y directions towards the destination point, $K, B \in \mathbb{R}^{2\times 2}$ are diagonal positive definite scaling matrices, and $\|e\|$ denote the L_∞ norm of the error term e, which represent the

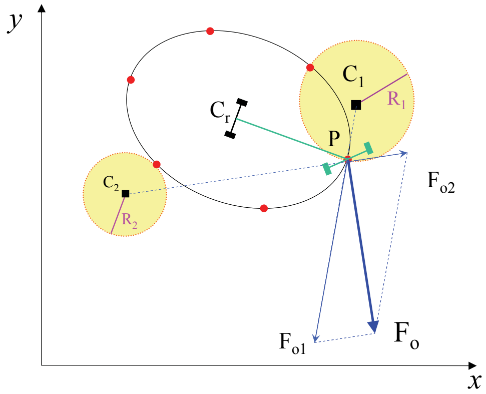


Fig. 2. The obstacle avoidance model of the path planner, where the avoidance forces F_{o_i} are applied from the obstacle centers C_i to the front axle center P of the reference robot.

difference between the desired front axle COM P_d and its current position P:

$$e = P_d - P = [x_{ed} - x_e,\ y_{ed} - y_e]^T, \quad \dot{e} = -[\dot{x}_e,\ \dot{y}_e]^T \tag{9}$$

The net effect of the steer force in (8) is to drag the reference robot front axle in a (PD) control like manner to the target, until the error terms $e, \dot{e}$ tend to zero.

2) Obstacle Detection and Avoidance: Path planners usually assume the global information of the robot task space is known priori [1], [2], [7]; however, in real world situations this is not usually possible. The WMRs can only access information from a limited part of their environment via their sensors, which we will refer to as the *Sensing Zone*. Because of the generally oval structure of the unicycle WMRs, the sensing zone is assumed to be ellipsoidal in our work. The sensing properties of a WMR equipped with entry level ON/OFF sensors has been depicted in Figure 2. In this configuration if an obstacle is encountered on the path of the WMR, the planner assumes that a circular blocking object is encountered in that direction. Two such obstacle estimates are depicted by the shaded circles in Figure 2. The center of the obstacles, C_i is calculated from the line segments starting from the mean point of the excited sensor group, tending to outside the sensing zone; and the radius (ie. the size) of each obstacle $R_i(t) \in \mathbb{R}^+$ from the number of the excited these sensors and the duration of their obstacle signals. The centers of the obstacles C_i are limited to be no further to the front than the steering axle center P, to ensure smooth route generation for obstacle estimates obtained from any sensor combination. In addition to these there are constant weights $w_i \in \mathbb{R}^+$ assigned to each of the associated force components according to the nearness of their sensors to P to emphasize avoidance in the direction of motion. The overall obstacle avoidance force F_o, is the vector sum of the force components $F_{o_i} = [F_{ox_i},\ F_{oy_i}]^T, \ \ i = 1, \ldots N$, from every concurrent obstacle center C_i to the front axle center of the reference robot P, as follows:

$$\begin{aligned} F_o(t) &= \textstyle\sum_{i=1}^{N} F_{o_i}(t) \\ &= \textstyle\sum_{i=1}^{N} w_i\, R_i(t)\, [P(t) - C_i(t))], \quad i = 1, \ldots, N \end{aligned} \tag{10}$$

where N is the number of current obstacles.

The ratio of each force component in the overall repulsion force F_o, increases according to the impact time of the related obstacle. However, the overall force is kept constant by normalizing the general obstacle avoidance force $\|F_o(t)\|$, to the force in direct steer mode $\|F_s(t)\|$. This ensures generation of bumpless reference velocities imperative for accurate nonholonomic control. The overall obstacle avoidance force $F_{on}(t)$ after this normalization is:

$$F_{on}(t) = \|F_s(t)\| \frac{F_o(t)}{\|F_o(t)\|}. \tag{11}$$

The general steering force of our path planner $F(t)$ resembles the avoidance force $F_o(t)$ when the system is in the vicinity of obstacles; and the direct steer force $F_s(t)$ when the WMR is sufficiently far from obstacles. By this way a computationally fast algorithm has been developed for collision free nonholonomic motion of WMRs.

Remark 1: If the obstacle avoidance forces F_{o_i} are ceased immediately after the encountered obstacles are out of sensor range, the path planner may start to switch between the forward steer and obstacle avoidance modes, which causes chattering in the overall steer force $F(t)$. To avoid this problem a virtual margin is added to the estimated radius of each obstacle, which prohibits exit from the avoidance mode for an $\triangle t$ seconds after the block is out of the sensing zone.

Remark 2: If the WMR orientation angle becomes very different from the target direction, the steering angle φ in Figure 1 may exceed $90°$. This imposes negative values for the linear velocity v_l, causing a loss of smoothness in the generated path at that location. As a way to overcome this problem, the cross-over from the obstacle repulsion mode to target attraction mode is implemented via a $1st$ order spline transition. The overall obstacle avoidance force function F_t in this period is defined as:

$$F_t = F_s \frac{(t - t_s)}{\delta t} + F_{on} \frac{(t_s + \delta t - t)}{\delta t} \tag{12}$$

where F_s and F_{on} are the front steering and normalized obstacle avoidance forces in (8) and (11) respectively. This cross-over phase is confined to $t \in [t_s, t_s + \delta t]$ interval, where t_s is the time instant when the obstacle is out of sensing zone, and δt is the duration of the transition.

Remark 3: It should be noted that when an obstacle interferes with the front sensor of the WMR, the choice of a left or a right turn is left to the option of the device user. A special input is integrated to the simulation model for this purpose. This flexibility reduces the complexity of the system; however, it may cause route deviations that increase mission times if not deadlocks for multi-robot path generation in complicated environments. For this reason further considerations has to be taken into account. We will cover them in the next subsection.

B. Path Planning for Multiple Robots

For practical applications of mobile robotics, it is very important to enable real-time path generation for large WMR

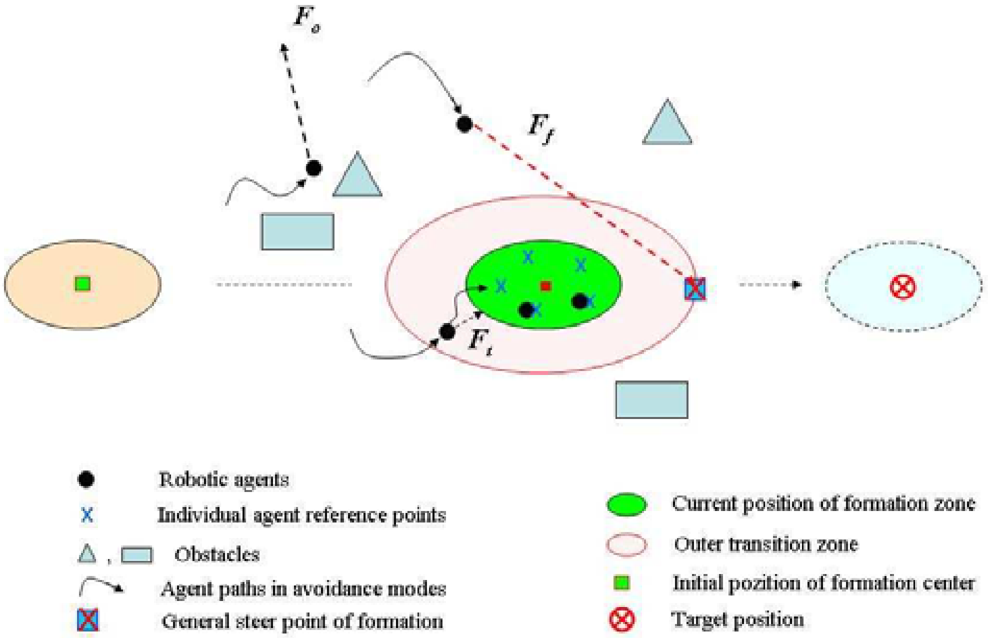


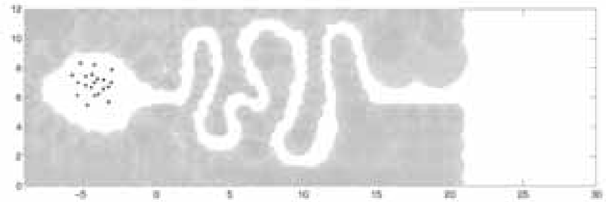
Fig. 3. The general multi-robot path planner outline: the green ellipse show the current position of the formation structure zones; the shaded red region encirculating the current formation is the robustness margin.

formations. In this section we will present a method for addressing this problem with robotic agents of limited sensing ability. Our method combines the single WMR planner introduced in the previous section with simple geometrical evaluations. This approach provides collision free path generation without necessitating any complicated high level approaches like swarm intelligence. The multi-robot planner could be based on routing of many decentralized WMRs from various initial positions in a certain region to corresponding terminal points in a desired region, while regarding each other as simple mobile blocks. This approach would be simple and is directly obtained by the application of the single WMR planner without any modification. However, there are two main problems associated with this simple technique, which may cause many flock elements not reaching their targets. These are (i) the front sensor ambiguity of Remark 3 inherited from the single robot planner; and (ii) the negative influence of various randomly scattered terminal points, both of which increase direction reversals, hindering not only the affected robot, but also the trailing traffic.

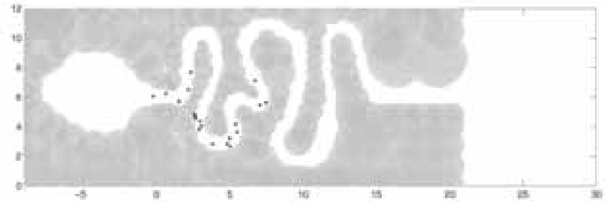
For this reason the simple geometrical framework described in Figure 3 is integrated with the proposed single WMR planner to achieve a simple and efficient flock technique. Accordingly the formation navigates nonholonomically from an initial region to a desired location with a steer force (8) similar to the bicycle reference robot model in Section IV-A. The formation velocity is lower than the individual agents, such that these WMRs can track their points in the formation by a modified steer force of the form:

$$F_s = K\frac{e_r}{\sqrt{\|e_r\|^2+\varepsilon}} + B\dot{e}_r, \tag{13}$$

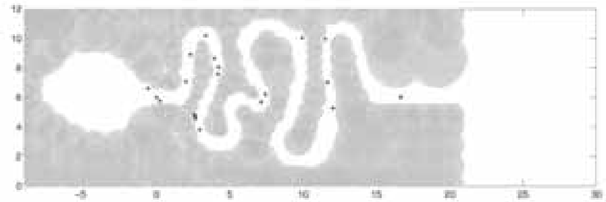
thereby navigating within the formation. In (13) the error function is the distance between the the current position of the agent front axle and its reference position within the formation, $e_r = [x_{er} - x_e,\ y_{er} - y_e]^T$. As the formation is mobile, the derivative of the error function is now $\dot{e}_r = [\dot{x}_{er} - \dot{x}_e,\ \dot{y}_{er} - \dot{y}_e]^T$ in parallel with tracking requirements. However, when an obstacle interferes with an agent in the formation a modified obstacle avoidance mechanism is introduced for robustness to possible WMR direction reversals. In such a case the steer force F, is equalized to $F = F_o$ for the deflection of the agent from the formation according to the single robot obstacle avoidance algorithm. When the encountered block is out of the sensor range, the robot is not re-directed directly to its reference point within the formation zone. Instead it is driven first towards the virtual leader like front axle center of the general formation, with the steer force equated to $F = F_f$. When the outer transition margin is reached, a simple first order spline similar to (12) is utilized to readjust the direction of the transitional WMR steer force $F = F_t$, smoothly from the virtual leading axle to the agent reference point in the formation. Thus at the outer perimeter of this zone, the agent drag force is $F_t = F_f$, which is smoothly changed to the steer-by-formation force F_s of (13) at the inner boundary. At this stage the actual formation zone has been reached, thus (provided no more obstacles are encountered) the steer force is preserved at $F = F_s$ to keep within the formation. The advantages of this multi stage convergence from the WMR pose at the end of obstacle avoidance mode to the specific agent position in the formation is two fold. Firstly, steering towards the front axle via $F = F_f$ reduces the risk of course reversals. Secondly the presence of outer margin reduces the risk of the agent to interrupt other members by first overtaking the slower moving formation and then move towards its frame in the opposite direction to the flock.



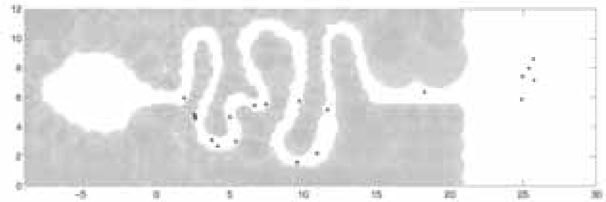
(a) Simulation Time t=0s



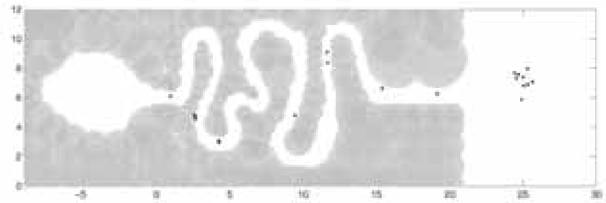
(b) t=200s



(c) t=400s

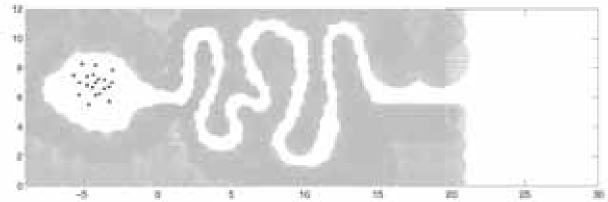


(d) t=600s

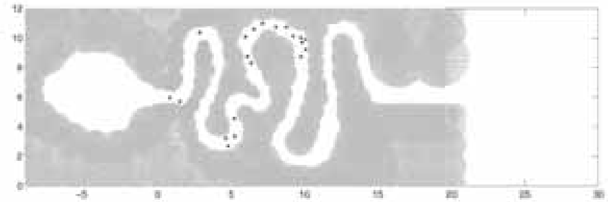


(e) t=800s

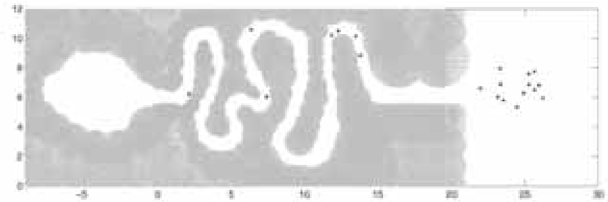
Fig. 4. The decentralized approach where the single robot planner of section IV-A is applied to individually to the members of the collection.



(a) Simulation Time t=0s



(b) t=200s



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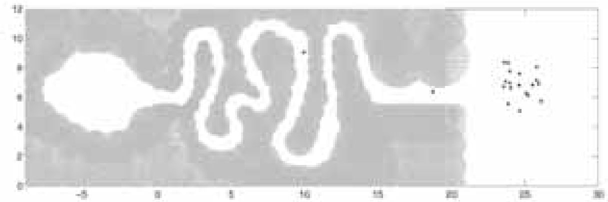
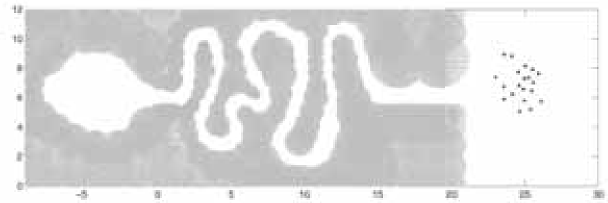


(d) t=600s



(e) t=800s

Fig. 5. The combined route generator with the individual WMR planner and the geometrical framework operating simultaneously.

V. SIMULATION RESULTS

The effectiveness of the robot collection path planner is verified by a demanding set of simulations, depicted in Figures 4 and 5. The simulation environment is a complex passage, with many sharp turns causing significant difficulty even for single WMR planners to overcome. In these figures, the shaded regions show the blocks, while the lighter areas denote the free areas. The aim of the simulations is the steering of a collection of 20 robots from the free area on the left side to the clearance on right.

Our simulations are carried out in Matlab©/Simulink© environments using C mex s-functions. The parameters of the simulation are as follows: for the obstacle sensors, we have utilized simple ON-OFF sensors located at the robot perimeter forming an ellipsoidal sensing zone. We set the number of these sensors to 6 in parallel with the sensing equipments of small WMR configurations. We have selected the parameters of the steering force function (8) and the major and minor axis lengths of the WMR sensing as:

$$K = diag(0.16, 0.16), \qquad B = diag(0.01, 0.01),$$
$$a = 22.5, \quad b = 20 \;\; [cm]. \tag{14}$$

in conjunction with the characteristics of small WMRs. The distance between the front and the back axles, L is equal to the major ellipsoidal axis a, as a copromise between maneuverability and limits of the steering mechanisms. The multi-robot geometrical structure is selected as a disc with the radius of $7a$ suitable for incorporating about 20 small robots with sufficient margin to prevent accumulating rate of path intersections. Likewise the outer margin of this ensemble is set to $10a$ in radius for efficient results. The durations of the δt and $\triangle t$ periods in Remarks 1 and 2 are 7.5 and 0.8 [seconds] respectively. The unified controller in [9] has been employed for the low level motion controller. As the single robot planner algorithm is compact and computationally efficient, and the geometrical multi-robot extensions preserves these properties, it was possible to achieve real time dynamical path generation and tracking control on a standard PC with a Pentium 4$^{©}$ CPU and 512MB RAM running Windows XP$^{©}$ operating system.

The simulations are depicted on Figures 4 and 5 for the simple steering algorithm and the formation based approach, respectively. As we can observe from these figures, there is about 30 meters of distance in a direct linear path between the initial and final positions of the group members. Thus the mission distance should be about twice as much, because of the sharp turns in the passage. Accordingly with a reference robot maximum linear velocity about 0.16m/s, the mission should last slightly less than 400 seconds assuming no direction reversals.

The two ensembles start at $t = 0$ seconds from the zone zone on the lhs of the figures 4(a) and 5(a), respectively. By the simulation time of 200 s (Figures 4(b) and 5(b)), it can be observed that the formation based member distribution is nearer to the terminal region. As depicted in figures 4(c) and 5(c), at about $t = 400$ s, more than half of the robots in the right hand side group have reached their targets, while not even a single robot are at that point for the other simple approach. Moreover some robots of the non-formation based system are deadlocked just before the $3rd$ turn of the passage. This shows the effectiveness of the proposed formation path planner in reaching the desired locations with little delay. At about $t = 600$ seconds only 2 WMRs are away from their targets for the geometrical formation based ensemble in Figure 5(d) and finally in less than 800 seconds all the robots reach their terminals (see. figure 5(e)). On the other hand when the simulation is over for the simpler approach on Figure 4(e), the group suffers 3 robots being deadlocked and 7 yet to reach their targets. Based on these simulations we can conclude that the geometrical formation framework is an effective approach, which can be used for transportation of large flocks, where the general topology of the workspace is unknown except few milestones. Moreover, our results are obtained at real-time from widely accessible, standard platforms and with path planner codes supporting ON/OFF sensors. These should bring the associated system costs significantly for possible implementations.

VI. CONCLUSIONS

We have presented a simple yet effective algorithm for the path generation of a large number of WMRs in obstacle ridden environments. Our method is based on the co-operation of a computationally low-cost single robot planner with a simple formation approach based on geometric evaluations rather than swarm intelligence. The overall method is computationally efficient and enables effective navigation in the virtually all cases of obstacle environments including tunnels of complicated geometry. Moreover, our algorithm works sufficiently even for ON/OFF sensors, thus is suitable for all types of WMRs including small, inexpensive ones with entry level sensing equipment. Simulation results are presented to illustrate the effectiveness of the proposed method. Future work will concentrate on the implementation of the associated theory.

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